2024

MATHEMATICS — HONOURS

Paper : SEC-3

(Linear Programming and Rectangular Games)

Full Marks : 75

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group – A

1. Answer ***any five*** questions : 3×5

(a) Solve graphically the LPP :

Maximize $Z = 5x_1 - 2x_2$

subject to $5x_1 + 6x_2 \geq 30$

$9x_1 - 2x_2 = 72$

$x_2 \leq 9,\ x_1, x_2 \geq 0$

Hence, identify the redundant equation, if there be any.

(b) Define convex set. Show that the set $X = \left\{(x, y) \in E^2 : xy \leq 1, x \geq 0, y \geq 0\right\}$ is not a convex set.

(c) What are the extreme points of the convex set $X = \left\{(x, y) \,\middle|\, |x| \leq 1, |y| \leq 1\right\}$? Give an example of a convex set which has no extreme point.

(d) For the system of equations

$$2x_1 - x_2 + 3x_3 = 3$$
$$-6x_1 + 3x_2 + 7x_3 = -9$$

show that (2, 1, 0) is not basic though it is feasible solution. Reduce the feasible solution to a basic feasible solution.

(e) Find a basic feasible solution of the system of equations :

$$x_1 + 2x_2 + x_3 = 4$$
$$2x_1 + x_2 + 5x_3 = 5$$

(f) Find the dual of the following linear programming problem :

Maximize $Z = 3x_1 + 2x_2$

subject to $2x_1 + x_2 \leq 5$

$x_1 + x_2 \leq 3$

$x_1, x_2 \geq 0$

Please Turn Over

(g) Find an initial basic feasible solution, by North-West method, of the following transportation problem :

	1	2	3	4	
X	4	2	5	3	6
Y	5	4	3	2	13
Z	1	4	6	5	9
	7	8	5	8	

(h) For what value of p, the game with the following pay-off matrix is strictly determinable?

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} p & 5 & 2 \\ -1 & p & -8 \\ -2 & 3 & p \end{bmatrix} \end{array}$$

Group – B

2. Answer ***any five*** questions :

(a) A company produces two types of leather belts, say type A and type B. Belt A is of a superior quality and belt B is of an inferior quality. Profits on the two types of belts are rupees 5 and rupees 4 per belt respectively. Each belt of type A requires twice as much time as required by a belt of type B. If all belts are of type B, the company could produce 1000 belts per day. Belt A requires a fancy buckle and only 400 buckles are available for this per day. For belts of type B only 700 buckles are available per day. Formulate the above problem as a linear programming problem. 6

(b) Solve graphically :

$$\begin{aligned} \text{Maximize} \quad & Z = 100x_1 + 50x_2 \\ \text{subject to} \quad & 4x_1 + 6x_2 \le 24 \\ & x_1 \le 4 \\ & x_2 \le \tfrac{4}{3},\ x_1, x_2 \ge 0. \end{aligned}$$

Is there any degeneracy in the problem? Justify your answer. 3+3

(c) Prove that x_1 and x_3 cannot be simultaneously considered as the basic variables of the set of equations :

$$2x_1 + x_2 - x_3 + 4x_4 = 6$$
$$6x_1 + 3x_2 - 3x_3 + x_4 = 10$$

Find all the basic solutions with x_1 as one basic variable. 6

(d) Prove that every basic feasible solution to a linear programming problem corresponds to an extreme point of the convex set of feasible solutions. 6

(e) Solve the following linear programming problem by simplex method : 6

$$\begin{aligned} \text{Minimize} \quad & Z = x_1 - 3x_2 + 2x_3 \\ \text{subject to} \quad & 2x_1 + 3x_2 - x_3 \le 7 \\ & 4x_3 - 2x_2 \le 12 \\ & 8x_1 - 4x_2 + 3x_3 \le 10 \\ & x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0. \end{aligned}$$

(f) Use Charnes Big M-method to show that the following LPP has unbounded solution : 6

Maximize $Z = 2x_1 + 3x_2 + x_3$

subject to
$$-3x_1 + 2x_2 + 3x_3 = 8$$
$$-3x_1 + 4x_2 + 2x_3 = 7$$
$$x_1, x_2, x_3 \geq 0$$

(g) Using Two-Phase Simplex method, show that the following LPP has no feasible solution : 6

Maximize $Z = 5x + 3y$

subject to $3x + 4y \geq 12$

$2x + y \leq 1,\ x, y \geq 0.$

(h) Show that the following LPP has unbounded solution : 6

Maximize $Z = -2x_1 + 5x_2$

subject to
$$4x_1 - 5x_2 \geq -20$$
$$x_1 + x_2 \geq 10$$
$$x_2 \geq 2$$
$$x_1, x_2 \geq 0.$$

Group – C

3. Answer ***any five*** questions :

(a) Find the dual of the following linear programming problem. Hence, prove that the dual has no optimal solution. 6

Minimize $Z = 3x_1 - 5x_2 + x_3$

subject to $x_1 - 2x_3 \geq 4$

$2x_1 - x_2 + x_3 \geq 2$

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0.$

(b) Suppose that a constraint in a given LPP is an equality. Prove that corresponding dual variable is unresctricted in sign. 6

(c) Solve the following transportation problem :

	D_1	D_2	D_3	D_4	a_i
O_1	5	3	6	4	30
O_2	3	4	7	8	15
O_3	9	6	5	8	15
b_j	10	25	18	7	

Is the solution unique? 5+1

Please Turn Over

(d) Solve the assignment problem with the following cost matrix : 6

	M_1	M_2	M_3	M_4	M_5
I	9	8	7	6	4
II	5	7	5	6	8
III	8	7	6	3	5
IV	8	5	4	9	3
V	6	7	6	8	5

(e) Solve the following travelling salesman problem : 6

		To			
		A	B	C	D
From	A	α	46	16	40
	B	41	α	50	40
	C	82	32	α	60
	D	40	40	36	α

(f) Reduce the following game to 2×2 game by dominance property and then solve it. 6

	B_1	B_2	B_3	B_4
A_1	3	2	4	0
A_2	3	4	2	4
A_3	4	2	4	0
A_4	0	4	0	8

(g) Solve the following 4×2 game graphically : 6

$$\begin{array}{cc} & \text{Player B} \\ \text{Player A} & \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 6 \\ 4 & 1 \\ 2 & 2 \\ -5 & 0 \end{bmatrix} \end{array}$$

(h) Solve the following game by linear programming technique : 6

		B	
	3	−1	−3
A	−3	3	−1
	−4	−3	3