

2025

MATHEMATICS — HONOURS

Paper : SEC-3

(Linear Programming and Rectangular Games)

Full Marks : 75

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.*

Group - A

1. Answer **any five** questions :

3×5

(a) Solve the following LPP by graphical method :

Minimize $Z = 4x_1 + x_2$

subject to $x_1 + 2x_2 \leq 3$

$4x_1 + 3x_2 = 6$

$3x_1 + x_2 \geq 3, x_1, x_2 \geq 0$

(b) Find a basic solution of

$x_1 + 4x_2 - x_3 = 3$

$5x_1 + 2x_2 + 3x_3 = 4$

with x_3 as non-basic variable.

(c) Prove that a hyperplane is a convex set.

(d) Prove that the set $X = \{(x, y) \in E^2 : y^2 \leq x\}$ is a convex set.

(e) Using simplex method, solve the following LPP :

Maximize $Z = 3x_1 + 9x_2$

subject to $2x_1 + 3x_2 \leq 6$

$2x_1 + 6x_2 \leq 6$

$x_1, x_2 \geq 0.$

Please Turn Over

(4995)

- (f) Find the dual of the following LPP :

$$\text{Maximize } Z = x_1 - x_2 + 3x_3 + 2x_4$$

$$\text{subject to } x_1 + x_2 \geq -1$$

$$x_1 - 3x_2 - x_3 \leq 7$$

$$x_1 + x_3 - 3x_4 = -2$$

$$x_1, x_2, x_3, x_4 \geq 0.$$

- (g) Obtain an initial basic feasible solution to the following Transportation problem using the matrix minima method :

	D_1	D_2	D_3	D_4	
O_1	1	2	3	4	6
O_2	4	3	2	0	8
O_3	0	2	2	1	10
	4	6	8	6	

- (h) Apply maximin and minimax principle to solve the game whose pay-off matrix is given below :

		Player B		
		B_1	B_2	B_3
Player A	A_1	-1	-2	8
	A_2	7	5	-1
	A_3	6	0	12

Group - B

2. Answer *any five* questions :

- (a) A furniture manufacturer plans to make only two products, chairs and tables, from his available resources. He has 400 board feet of wood and 450 man-hours. A chair requires 5 board feet of wood and 10 man-hours and yields a profit of ₹ 140.00, while one table requires 20 board feet of wood and 15 man-hours and yields a profit of ₹ 300. Pose a linear programming problem so that he can maximize his profit. 6

- (b) (i) Is $(2, 0, 0, 1, 0)$ a basic feasible solution of the system of equations

$$2x_1 + 2x_2 - x_3 + x_4 + 5x_5 = 5$$

$$4x_1 + 3x_2 - 3x_3 + 2x_4 - 3x_5 = 10?$$

Justify your answer.

- (ii) $(2, 4, 5)$ is a feasible solution of the set of equations

$$2x_1 - x_2 + 2x_3 = 10$$

$$x_1 + 4x_2 = 18$$

Reduce it to a basic feasible solution of the set.

2+4

- (c) Prove that the objective function of a linear programming problem attains its optimal value (if it exists) at an extreme point of the convex set of feasible solutions. 6

- (d) What do you mean by degenerate basic feasible solution?

Solve graphically to show that the LPP

$$\text{Maximize } Z = 3x_1 + 9x_2$$

$$\text{subject to } x_1 + 4x_2 \leq 8$$

$$x_1 + 2x_2 \leq 4, x_1, x_2 \geq 0$$

admits of a degenerate optimal basic feasible solution.

2+4

- (e) Solve the following linear programming problem by simplex method : 6

$$\text{Maximize } Z = 3x_1 + 5x_2 + 2x_3$$

$$\text{subject to } x_1 + x_2 + 2x_3 \leq 430$$

$$3x_1 + x_2 \leq 460$$

$$x_1 + 4x_3 \leq 420$$

$$x_1, x_2, x_3 \geq 0.$$

- (f) Use Big M method to solve the following LPP : 6

$$\text{Maximize } Z = 2x_1 + x_2 + 3x_3$$

$$\text{subject to } x_1 + x_2 + 2x_3 \leq 5$$

$$2x_1 + 3x_2 + 4x_3 = 12$$

$$x_1, x_2, x_3 \geq 0.$$

- (g) Solve the following LPP by two-phase method : 6

$$\text{Minimize } Z = 4x_1 + x_2$$

$$\text{subject to } 3x_1 + x_2 = 3$$

$$4x_1 + 3x_2 \geq 6$$

$$x_1 + 2x_2 \leq 4$$

$$x_1, x_2 \geq 0.$$

- (h) (i) Show that the following LPP has no feasible solution :

$$\text{Maximize } Z = 3x_1 + 2x_2$$

$$\text{subject to } 2x_1 + x_2 \leq 2$$

$$3x_1 + 4x_2 \geq 12$$

$$x_1, x_2 \geq 0.$$

- (ii) Prove that the set of all feasible solutions of a linear programming problem is a convex set.

3+3

Please Turn Over

(4995)

Group - C

3. Answer **any five** questions :

(a) Find the optimal solution of the following LPP by solving its dual :

6

$$\text{Minimize } Z = 3x_1 + 2x_2$$

$$\text{subject to } 7x_1 + 2x_2 \geq 30$$

$$5x_1 + 4x_2 \geq 20$$

$$2x_1 + 8x_2 \geq 16$$

$$x_1, x_2 \geq 0.$$

(b) Solve the following transportation problem by Vogel's approximation method :

6

	D_1	D_2	D_3	D_4	Available
O_1	15	20	13	21	30
O_2	22	15	19	14	12
O_3	16	12	14	31	18
O_4	24	23	15	30	10
Demand	25	20	15	10	

(c) Solve the following travelling salesman problem :

6

	A	B	C	D	E
A	∞	7	6	8	4
B	7	∞	8	5	6
C	6	8	∞	9	7
D	8	5	9	∞	8
E	4	6	7	8	∞

(d) (i) State the maximin or minimax theorem.

(ii) Find the value of the following 2×2 game algebraically by using mixed strategies :

		Player B	
		B_1	B_2
Player A	A_1	2	3
	A_2	4	-1

2+4

- (e) Solve the game problem using the dominance property :

6

		Player B				
Player A		0	2	0	1	1
		4	2	0	2	1
		4	3	1	3	2
		4	3	4	-1	2

- (f) Solve the following
- 2×4
- game graphically :

6

		Player B			
		B_1	B_2	B_3	B_4
Player A	A_1	6	5	2	3
	A_2	1	2	6	3

- (g) Solve the following game problem by using linear programming method :

6

		Player B		
		B_1	B_2	B_3
Player A	A_1	1	0	-2
	A_2	0	3	-2

- (h) If
- X
- be any feasible solution to a primal problem and
- V
- be any feasible solution to its dual problem, then prove that
- $CX \leq b'V$
- , where
- C
- and
- b
- are respectively the cost vector and the requirement vector of the primal problem.

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