

2025

MATHEMATICS — HONOURS

Paper : DSCC-2

(Basic Algebra)

Full Marks : 75

*The figures in the margin indicate full marks.**Candidates are required to give their answers in their own words as far as practicable.**Notations and Symbols have their usual meaning.*

Group - A

1. Answer **any two** questions :

2½×2

- (a) Find the *modulus* and principal amplitude of the complex number $Z = 1 + i \tan \theta$, $\frac{\pi}{2} < \theta < \pi$.
- (b) For positive real numbers a, b, c , find the least value of $a^{-1} + b^{-1} + c^{-1}$ if $a + b + c = 2020$.
- (c) Find an equation whose roots are reciprocal of the roots of the equation $x^3 + 5x^2 - 8x + 10 = 0$.

2. Answer **any four** questions :

5×4

- (a) If $x = \log \tan \left(\frac{\pi}{4} + \frac{\theta}{2} \right)$, where θ is real, then prove that $\theta = -i \operatorname{Log} \tan \left(\frac{\pi}{4} + i \frac{x}{2} \right)$.
- (b) If $\sin(\theta + i\phi) = \tan \beta + i \sec \beta$, then prove that $\cos 2\theta \cosh 2\phi = 3$.
- (c) If x, y and z are positive real numbers and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$, then show that the least value of $x^2 + y^2 + z^2$ is 27.
- (d) If α, β, γ are the roots of the equation $x^3 + qx + r = 0$, then find the equation whose roots are $\frac{\beta + \gamma}{\alpha^2}, \frac{\gamma + \alpha}{\beta^2}, \frac{\alpha + \beta}{\gamma^2}$.
- (e) Solve the equation, $x^4 + 4x^3 - 6x^2 + 20x + 8 = 0$ by Ferrari's method.
- (f) By Sturm method prove that the roots of the equation $x^3 - (a^2 + b^2 + c^2)x - 2abc = 0$ are all real.

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Group - B

3. Answer **any two** questions :

2½×2

- (a) If S be a finite set with n elements, then find the number of reflexive relations that can be defined on S .
- (b) Find the remainder when $6(7)^{32} + 7(9)^{45}$ is divided by 4.
- (c) For any prime p and for any two integers a, b if $p|ab$, then show that either $p|a$ or $p|b$.

4. Answer **any four** questions :

- (a) Let S be the set of all positive divisors of 36. On S define a relation ρ by apb if and only if $a|b$; for $a, b \in S$. Prove that (S, ρ) is a poset. Is it a linear ordered set? Justify your answer. 3+2
- (b) (i) Show that an equivalence relation on a non-empty set induces a partition on the set.
- (ii) A relation ρ on $\mathbb{Z}$ is defined by apb if and only if $ab \geq 0$. Is ρ an equivalence relation on $\mathbb{Z}$? Justify. 3+2
- (c) (i) If $\gcd(a, b) = 1$, then show that $\gcd(a + b, a - b) = 1$ or 2.
- (ii) Prove that the number of prime is infinite. 2+3
- (d) Solve : $x \equiv 2 \pmod{3}$; $x \equiv 3 \pmod{5}$; $x \equiv 4 \pmod{7}$. 5
- (e) If $n = p_1^{a_1} p_2^{a_2} \dots p_r^{a_r}$ and $p_1, p_2, \dots, p_r$ are prime to each other, then prove that

$$\phi(n) = n \left(1 - \frac{1}{p_1}\right) \left(1 - \frac{1}{p_2}\right) \dots \left(1 - \frac{1}{p_r}\right),$$

where ϕ is Euler's phi function.

5

- (f) Find $\sigma(900)$ and $\tau(900)$.

5

Group - C

5. Answer **any two** questions :

2½×2

- (a) Express the vector $(1, 1, 1)$ as a linear combination of the vectors $(2, 1, 1)$ and $(1, 2, 2)$ in $\mathbb{R}^3$.
- (b) Identify the free variable of the system of equation

$$2x + y + 12z = 1$$

$$x + 2y + 9z = -1$$

- (c) Find the value of k for which the system of equations

$$x + y + z = 0$$

$$y + 2z = 0$$

$$kx + z = 0$$

has more than one solution.

6. Answer **any four** questions :

5×4

(a) Determine the conditions for which the system of equations given below has

(i) only one solution, (ii) no solution and (iii) many solutions.

$$x + 2y + z = 1$$

$$2x + y + 3z = b$$

$$x + ay + 3z = b + 1$$

(b) Find a row-reduced echelon form of the matrix

$$\begin{pmatrix} 0 & 0 & 2 & 2 & 0 \\ 1 & 3 & 2 & 4 & 1 \\ 2 & 6 & 2 & 6 & 2 \\ 3 & 9 & 1 & 10 & 6 \end{pmatrix}$$

and find its rank.

(c) If A is an $n \times n$ invertible matrix, then prove that A has n pivot positions.

(d) Solve the system of equations

$$y + z = a, x + z = b, x + y = c,$$

and use the solution to find the inverse of the matrix.

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

(e) Consider the vectors $(1, 2), (2, 3), (3, 4), (4, 5)$ in $\mathbb{R}^2$. Let P be the set spanned by the vectors $(1, 2), (2, 3)$ and Q be the set spanned by the vectors $(3, 4), (4, 5)$. Examine whether $P = Q$.

(f) Find all real λ for which the rank of the matrix A is 2, when

$$A = \begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & 5 & 3 & \lambda \\ 1 & 1 & 6 & \lambda+1 \end{pmatrix}$$

Hence, solve the following system of equations for that value(s) of λ :

$$x + 2y + 3z = 1$$

$$2x + 5y + 3z = \lambda$$

$$x + y + 6z = \lambda + 1.$$